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GASDYNAMIC CHARACTERISTICS OF FLOWS IN PROBLEMS OF THE LAUNCHING OF INCOMPRESSIBLE PLATES BY DETONATION PRODUCTS

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UDC 534.222.2

There has been growing interest lately in analytical methods for the solution of one-dimensional gasdynamic problems involving the launching of incompressible plates [1-7]. This preoccupation stems from the relative simplicity of theoretical investigations and the feasibility of obtaining analytical solutions, identifying the principal gasdynamic characteristics of the generated flows, and both predicting and optimizing the gasdynamic possibilities of the analyzed launching configurations when the flow of detonation products is assumed to be isentropic and the launched plate is assumed to be incompressible.

The majority of the flow regions studied in [1-6] represent centered rarefaction waves, except that the centers of the waves can either be a part of or lie outside the analyzed region of flow of the detonation products, depending on the initial and boundary conditions of the problem. In this case the solutions can have regions where the families of rectilinear $(u \pm c)$-characteristics do not have a unique point of intersection (wave center), but form an envelope, which lies outside the investigated wave region. A similar situation arises, e.g., in the convergence of the characteristics in a compression wave. The difference is that the envelope in the latter case is situated in the wave region. An equation has been derived [7] for the envelope of the family of characteristics of a simple compression wave. In the present article we investigate a procedure for determining the envelope of the family of characteristics of a rarefaction wave. We analyze a method based on the equation for the envelope of a rarefaction wave for solving the planar one-dimensional isentropic gasdynamic equations.

One of the possible situations, which is associated with the occurrence of an envelope in the problem of the launching of a plate by detonation products is depicted in Fig. 1, which shows the trajectory of the plate 1, the envelope of the rarefaction wave 2, and the analyzed region I of flow of the detonation products. In this case the envelope is formed

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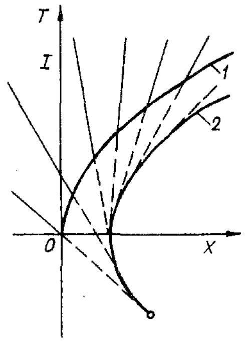

Fig. 1

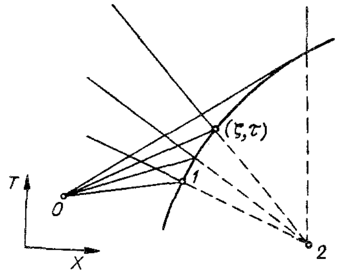

Fig. 2

by the family of (u − c)-characteristics. Such a flow region occurs, in particular, in the launching of a plate by the products of detonation of a high-explosive charge in which the detonation regime involves variable energy release at the wavefront (see, e.g., [8]).

We determine the equation for the envelope of the given family of characteristics. We seek a solution in dimensionless variables, using the thickness l of the explosive charge layer, the Chapman–Jouguet detonation rate, the time for a Chapman–Jouguet detonation wave to traverse a charge layer of thickness l and the mass of the charge layer (on unit area) as the reference units. We denote the corresponding dimensionless variables and parameters by X, T, U, C, and M.

We use the general solution of the one-dimensional isentropic gasdynamic equations. Introducing the notation β = U − C, we write the solution for the function β in the form

$$X = \beta T + F(\beta). \tag{1}$$

Differentiating Eq. (1) with respect to β and making use of the fact that $dX/d\beta = 0$ on the envelope, we obtain

$$T = -dF(\beta)/d\beta. \tag{2}$$

Substituting this expression in the general solution (1), we find

$$X = F(\beta) - \beta dF(\beta)/d\beta. \tag{3}$$

Equations (2) and (3) describe the envelope of the family of (u − c)-characteristics on the plane of the space–time variables X, T in parametric form (with parameter β).

We consider the motion of an incompressible plate in the region of the rarefaction wave. The law governing the motion of the plate is assumed to be known: $X = \zeta(\tau)$. We write the equation for the trajectories of the family of rectilinear β-characteristics beginning at the plate:

$$X = \zeta(\tau) + \beta(T - \tau). \tag{4}$$

According to Eq. (4), the arbitrary function $F(\beta) = \zeta(\tau) - \beta\tau$ in the general solution (1); substituting this function in Eq. (2) and taking Eq. (4) into account, we obtain

$$T = \tau + \beta\frac{d\tau}{d\beta} - \frac{d\zeta}{d\tau}\frac{d\tau}{d\beta} = \tau + \frac{\beta - \dot{\zeta}}{\dot{\beta}}. \tag{5}$$

We use this relation to obtain from (4)

$$X = \zeta + \beta(\beta - \dot{\zeta})/\dot{\beta}. \tag{6}$$

The dot is used everywhere to signify differentiation with respect to the variable τ.

Representing $\dot{\beta}$ in the form

$$\dot{\beta} = \frac{d\beta}{d\tau} = \frac{d\beta}{dU}\frac{dU}{d\tau} = \lambda\ddot{\zeta}, \tag{7}$$

we write Eqs. (5) and (6) as follows:

$$T=\tau+(\beta-\dot{\zeta})/(\lambda\ddot{\zeta}),\ X=\zeta+\beta(\beta-\dot{\zeta})/(\lambda\ddot{\zeta}). \tag{8}$$

All the quantities on the right-hand sides of the first and second equations of the system (8) are known functions of τ. Allowing for the fact that $\dot{\zeta}$ = U and $\beta-\dot{\zeta}=-C$, we write the equation for the envelope of the rarefaction wave in the form

$$T(\tau)=\tau-C(\tau)/(\lambda\ddot{\zeta}(\tau)),\ X(\tau)=\zeta(\tau)-\beta(\tau)C(\tau)/(\lambda\ddot{\zeta}(\tau)). \tag{9}$$

An analysis of the system (9) shows that $C > 0$, $\ddot{\zeta} > 0$, and $\lambda > 0$ on the contact surface of the launched plate, so that always $T < \tau$ and $\zeta < X$, because $\beta < 0$. At the initial time $\tau = 0$ we have $\zeta = 0$ and $T < 0$, but $X > 0$. It follows from this result that the envelope is formed outside the analyzed flow region.

It is particularly important to note that λ = const = 2 in the important practical case of an incompressible plate moving in the region of a simple rarefaction wave. Indeed, because of the relation $U - C = 2U - J_+$ and the constancy of the Riemann invariant J_+, it follows from Eq. (7) that $\lambda = d\beta/dU = d(2U - J_+)/dU = 2$.

As an example, we consider the procedure for obtaining the equation of the envelope of a rarefaction wave in the uniformly accelerated motion ($d^2X/dT^2 = a$ = const) of an incompressible plate propelled by detonation products. We use the parametric equations (9). The parameters of the motion of the plate in this case are $\zeta = a\tau^2/2$, $U = d\zeta/d\tau = a\tau$, $C = 1/2$; on the basis of the latter equation, λ = const = 1. Substituting the resulting quantities in Eqs. (9), we obtain

$$T=\tau-1/(2a),\ X=\tau(a\tau-1)/2+1/(4a). \tag{10}$$

Equations (10) describe the envelope of the family of rectilinear (u − c)-characteristics of the investigated region of the flow of the detonation products in parametric form. Eliminating the parameter τ, we write the envelope equation in the form $X = aT^2/2 + 1/(8a)$. The coordinates of the initial point of the envelope are determined by substituting the initial conditions $\tau = 0$, $\zeta = 0$ in Eqs. (10): $T_0 = -1/(2a)$, $X_0 = 1/(4a)$.

On the plane of the space–time variables X, T the envelope of the rarefaction wave for the uniformly accelerated motion of an incompressible plate has the form shown in Fig. 1 (curve 2). The envelope has an extremely diverse shape, depending on the law governing the motion of the plate; the envelope degenerates to a point for a certain law of motion of the plate. The characteristics converge to a single point in this case, namely the center of the rarefaction wave.

Using the envelope equation, we can show that if the rarefaction wave incident on the incompressible plate is centered, the reflected rarefaction wave will also be centered. We consider the planar one-dimensional motion of an incompressible plate in the region of a centered rarefaction wave (Fig. 2). We obtain the parameters of the motion of the plate from the solution of the system of equations

$$dU/dT=\eta C^3,\ dX/dT=U \tag{11}$$

(η = 16/27M, where M is the relative mass of the launched plate). Assuming that the center of the incident rarefaction wave on the plate is situated at the point with coordinates X_0, T_0 (point O in Fig. 2), we represent the family of rectilinear (u + c)-characteristics in the form

$$U+C=(X-X_0)/(T-T_0). \tag{12}$$

Differentiating this equation with respect to T and transforming the result on the basis of Eq. (11), for C we obtain the Bernoulli equation, which has the total integral

$$C=[a_1(T-T_0)^2-2\eta(T-T_0)]^{-1/2}. \tag{13}$$

The law of motion of the plate is determined by integrating the second equation of the system (11) after the substitution of relations (12) and (13) on its right-hand side. The resulting expression has the form

$$(X-X_0)/(T-T_0)=a_2-1/[\eta C(T-T_0)]. \tag{14}$$

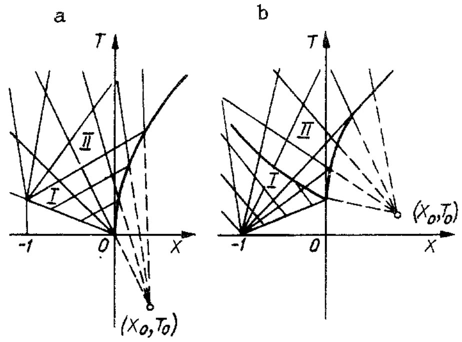

Fig. 3

The constants of integration a_1 and a_2 can be evaluated if the equation for the trajectory of the first (u + c)-characteristic arriving at the plate (at point 1 in Fig. 2) is known. Suppose that the slope of the given characteristic is α_1; then

$$a_1 = \frac{1}{C_1^2 (T_1 - T_0)^2} + \frac{2\eta}{T_1 - T_0}, \quad a_2 = \alpha_1 + \frac{1}{\eta C_1 (T_1 - T_0)}.$$

From Eq. (12) we find the velocity of the plate

$$U = (X - X_0)/(T - T_0) - C. \tag{15}$$

Consequently, if the center X_0, T_0 of the rarefaction wave and the coordinates of the point of intersection of the plate trajectory with the α_1-characteristic are known, the subsequent motion of the plate is completely determined.

We now substantiate the assertion that the reflected rarefaction wave is centered if the rarefaction wave incident on the incompressible plate is centered, using the general solution of the one-dimensional isentropic gasdynamic equations. Expressing the arbitrary function F(U − C) according to Eq. (1) with allowance for (13)-(15), we obtain

$$F(U - C) = X_0 - a_2 T_0 + \frac{(a_1 T_0 + 2\eta)(T - T_0)}{\eta \sqrt{a_1 (T - T_0)^2 - 2\eta (T - T_0)}}.$$

Since, according to Eqs. (13) and (15),

$$U - C = a_2 - \frac{a_1 (T - T_0)}{\eta \sqrt{a_1 (T - T_0)^2 - 2\eta (T - T_0)}},$$

the arbitrary function can be written in the form

$$F(U - C) = X_0 + 2\eta a_2/a_1 - (T_0 + 2\eta/a_1)(U - C),$$

and Eq. (1) can be written in the form

$$X = X_0 + 2\eta a_2/a_1 + (T - T_0 - 2\eta/a_1)(U - C). \tag{16}$$

Equation (16) is the solution for a centered rarefaction wave with center coordinates

$$X = X_0 + 2\eta a_2/a_1, \quad T = T_0 + 2\eta/a_1 \tag{17}$$

(point 2 in Fig. 2). Thus, the rarefaction wave reflected from the incompressible plate is also centered. It follows from Eq. (17) that the constant a_2 characterizes the slope of the line joining the centers of the incident and reflected rarefaction waves.

We substantiate the assertion, using the envelope equation. We consider the reflected rectilinear (u − c)-characteristic emanating from the point with coordinates ζ, τ on the line describing the plate trajectory on the X-T plane (Fig. 2). The trajectory of the characteristic is described by the equation

$$X - \zeta = (U - C)(T - \tau). \tag{18}$$

Allowing for the fact that ζ, U, and C are known functions of τ, we represent expression (18) by the equation $\Phi(X, T, \tau) = 0$, which contains the arbitrary parameter τ. Inasmuch as τ varies along the plate trajectory, the latter equation can be regarded as the equation for the trajectories of the family of (u − c)-characteristics of the region of flow of the detonation products.

We know that if a one-parameter family of curves has an envelope, the equation for the latter is determined from the solution of the system

$$\Phi(X,\ T,\ \tau)=0,\ \partial\Phi(X,\ T,\ \tau)/\partial\tau=0, \tag{19}$$

and the equation for the discriminant curve $F(X, T) = 0$ is determined by eliminating the parameter τ from the system (19). Returning to Eq. (18) and making use of expressions (13)-(15), in which the variable T must be replaced by the variable τ, we find

$$\begin{gathered} U(\tau)-C(\tau)=a_2-a_1C(\tau)(\tau-T_0)/\eta, \\ \zeta(\tau)-\tau[U(\tau)-C(\tau)]=X_0-a_2T_0+(a_1T_0+2\eta)C(\tau)(\tau-T_0)/\eta. \end{gathered} \tag{20}$$

Adopting the function $f(\tau) = C(\tau)(\tau - T_0)$ instead of τ as the arbitrary parameter in the system (20), we obtain from Eqs. (18) and (20)

$$\begin{gathered} \Phi(X,T,f)\equiv X-\left[a_2-\frac{a_1f(\tau)}{\eta}\right]T-\left[X_0-a_2T_0+\frac{a_1T_0+2\eta}{\eta}f(\tau)\right]=0, \\ \partial\Phi(X,\ T,\ f)/\partial f\equiv 2+a_1(T-T_0)/\eta=0. \end{gathered} \tag{21}$$

The solution of the system (21) has the form $X = X_0+2\eta a_2/a_1$, $T=T_0+2\eta/a_1$, which coincides with the solution (17). Thus, the envelope of the family of (u − c)-characteristics degenerates to a point. Consequently, the reflected rarefaction wave is centered.

The envelope equation can also be used to find the solutions of the planar one-dimensional isentropic gasdynamic equations. We analyze the method used to find the solutions for centered rarefaction waves in application to problems in the launching of an incompressible plate by the products of detonation of a high-explosive charge [1-4], in which detonation is initiated either on the contact surface of the launched plate (Fig. 3a) or on the free surface of the charge (Fig. 3b). The analysis is carried out for the region II of flow of the detonation products.

In the investigated flow region the continuations of the (u − c)-characteristics emanating from the contact surface of the launched plate intersect at a single point. The coordinates of the intersection point, which is the center of the wave, lie outside the analyzed flow region and can be determined from the envelope equation (9) with allowance for the boundary conditions on the contact surface of the plate for the analyzed launching configuration.

We consider the region II of flow of the detonation products in the launching problem shown in Fig. 3a. In this case $\lambda = 2$, since region II is the region of a simple wave, in which case the Riemann invariant is constant: $J_+ = 1/2$.

From the equation of motion of the plate

$$\ddot{\zeta}=dU/d\tau=\eta C^3 \tag{22}$$

in conjunction with the relation $dU/d\tau = d(J_+ - C)/d\tau = -dC/d\tau$ we obtain $dC/d\tau = -\eta C^3$. Integrating the latter equation subject to the initial conditions $\tau = 0$, $C = 1/2$, we have

$$\tau=1/(2\eta C^2)-2/\eta. \tag{23}$$

The coordinate T_0 of the center of the rarefaction wave is determined by substituting expressions (22) and (23) in the first equation of the system (9):

$$T_0=-2/\eta. \tag{24}$$

To find the spatial coordinate X_0 of the center of the rarefaction wave, we write the solution for the family of (u − c)-characteristics

$$(X-X_0)/(T-T_0)=U-C, \tag{25}$$

where T_0 is given by Eq. (24). Allowing for the fact that the slope of the first (u − c)-characteristic of the region II of flow of the detonation products $U - C = -1/2$ and substituting $T = 0$, $X = 0$ from Eq. (25), we find $X_0 = 1/\eta$. Substituting the values of X_0 and T_0 in Eq. (25) and transforming the result to the form $X = (U - C)T + 4U/\eta$, we obtain a particular solution describing the flow in the region of the centered rarefaction wave.

We now consider the region II of flow of the detonation products in the launching problem represented by Fig. 3b. It is more convenient to use Eqs. (5) and (6) to determine the coordinates of the point of intersection of the family of (u − c)-characteristics.

We write the solution for the family of (u − c)-characteristics in II in the form

$$\beta = U - C = 2U - (U + C) = 2U - X/\tau. \tag{26}$$

Differentiating this equation with respect to τ and taking expression (22) into account, we obtain

$$\dot{\beta} = 2\eta C^3 + C/\tau. \tag{27}$$

Making use of the relation $\dot{\zeta} = U$ and substituting expressions (26) and (27) in Eq. (5), we have

$$T = \tau[1 - 1/(1 + 2\eta C^2\tau)]. \tag{28}$$

To determine the coordinate T_0 of the center of the rarefaction wave, it is necessary to express the sound velocity C in terms of the variable τ. The determination of the function $C(\tau)$ is reducible to the solution of the differential equation (22); this equation, in turn, is reduced on the basis of the relation

$$\frac{dU}{d\tau} = \frac{d(U + C - C)}{d\tau} = \frac{d(X/\tau - C)}{d\tau} = -\frac{C}{\tau} - \frac{dC}{d\tau}$$

to the Bernoulli equation, whose partial integral with the initial conditions $\tau = 1$, $C = 1$ has the form $C^2 = [(1 + 2\eta)\tau^2 - 2\eta\tau]^{-1}$. After this expression is substituted in relation (28), we have

$$T_0 = 2\eta/(1 + 2\eta). \tag{29}$$

To find the coordinate X_0 of the center of the rarefaction wave, we use Eq. (25), in which T_0 is given by Eq. (29). Allowing for the fact that the slope of the first (u − c)-characteristic of region II $U - C = -1$ in this case and substituting $T = 1$, $X = 0$, from Eq. (25) we obtain $X_0 = 1/(1 + 2\eta)$. Substituting the values of the parameters X_0 and T_0 in Eq. (25) and transforming the expression to the form $X = (U - C)T + [1 - 2\eta(U - C)]/(1 + 2\eta)$, we arrive at the general solution for the family of (u − c)-characteristics of the region II of flow of the detonation products.

Thus, the equation for the envelope of the family of rectilinear characteristics of a rarefaction wave can be used to find solutions of the one-dimensional isentropic gasdynamic equations in problems of the launching of an incompressible plate by detonation products. The relative simplicity of the solutions given for centered rarefaction waves is attributable to the fact that these solutions are functions of the complex argument $(X - X_0)/(T - T_0)$, which combines the basic arguments X and T, i.e., the possibility of obtaining a solution in the analyzed flow region is associated with the determination of the coordinates X_0, T_0 of the center of the wave. This fact permits the above-described method to be used also for finding solutions in more complicated gasdynamic launching problems, since the majority of the flow regions in problems of this kind are regions of centered rarefaction waves.

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SHOCK COMPRESSION OF POROUS MATERIALS

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Porous materials (sintered metal powders and foamed plastics) are complex mechanical structures. When loaded by shock waves to the point where their strength properties manifest themselves they display certain characteristic peculiarities.

The study of foamed polystyrene performed in [1] showed the presence of two steady-state shock waves followed by a nonsteady-state plastic compression wave. Upon motion of the two-wave system through the specimen the shock wave amplitude and velocity remain constant, depending solely on the relative density of the polystyrene. A similar complex structure with two steady-state shock waves has been observed in specimens of sintered copper powder [2, 3].

Using a unified methodological approach the present study will analyze experimental results for foamed polystyrene [1] and a number of sintered metals: copper, aluminum, tungsten, and beryllium [4-20]. Various methods exist for deriving analytical expressions to describe the mechanical characteristics of the porous material as functions of the relative density d, which is equal to the ratio of the porous material density to the density of the matrix material. For example, use has been made of theoretical studies of composite materials containing inclusions of close to spherical form, since vanishing of the elastic characteristics of the inclusions permits extending the results of such studies to porous materials. In [4-9] the elastic characteristics of composition materials were studied using the variation principles of the linear theory of elasticity. Estimates of elastic moduli were obtained using various models of the porous material structure. Of those studies we must take special note of [9], which obtained analytical expressions for the shear modulus and volume compression of porous materials, the use of which permits one to determine speed of propagation of oscillations in an infinite porous medium.

In [10] the dependence of the relative density of parts formed from metal powder upon pressing pressure was presented in the form of a power function. This approach was used later in [11, 12]. Various mechanisms for cell wall deformation in foamed plastic dependent on relative density were noted in [13].

We will represent the isotropic porous body in the form of a set of elementary cells, the boundaries of which are shown by dashed lines in Fig. 1. Such a representation is most obvious for bodies of the foamed plastic type. Sintered powders will be considered to consist of particles having acoustical contact and forming cells of the type shown in Fig. 1. Elastic perturbations propagate along some winding path formed by the elementary cell surfaces (solid line of Fig. 1). We introduce the following notation: v_0, mean size of an individual pore; N, number of pores per unit volume of the porous body; v_1, mean volume of an elementary cell of the porous body; v_2, volume of solid material in the porous body elementary cell. We will note that $v_2 = \lim v_1$ as $v_0 \to 0$. If we represent the porous body in the form of a cube of unit volume, then $v_0 = (1 - d)/N$, $v_1 = 1/N$, $v_2 = (1 - Nv_0)/N = d/N$.

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